Solutions to JEE Main Home Practice Test - 11 | JEE - 2024

PHYSICS

SECTION-1

1.(A) (i) $[LC]=\left[\frac{L}{R}RC\right]=T^2$

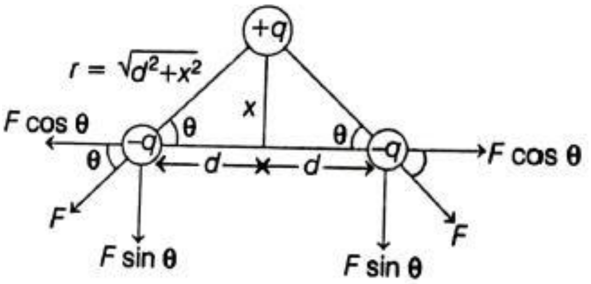
(ii) $[LR]=\left[\frac{L}{R}R^2\right]=T[R^2]=T\left[\frac{V}{I}\right]^2=T\left[\frac{W}{QI}\right]^2$

$=T\left(\frac{W}{I^2t}\right)^2=T\left(\frac{ML^2T^{-2}}{A^2T}\right)^2=M^2L^4T^{-5}A^{-4}$

(iii) $[H]=\left[\frac{\text{Heat}}{\text{Mass}}\right]=\frac{ML^2T^{-2}}{M}=L^2T^{-2}$

(iv) $[S]=\left[\frac{\text{Heat}}{\text{Mass}\times\text{Temperature}}\right]=\frac{ML^2T^{-2}}{MK}=L^2T^{-2}K^{-1}$

2.(C) The arrangement of charges is shown below



As we know that

Coulomb's force between two charges

i.e., q_1 and q_2

$$F=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{(d^2+x^2)} \quad \ldots\text{(i)}$$

Here, $q_1=q_2=q$

Force in SHM, $F=m\omega^2x$ …(ii)

Since, in order to have SHM +q should move downwards and force responsible for this will be only

$F'=F\sin\theta+F\sin\theta=2F\sin\theta$ …(iii)

Using equations (ii) and (iii), we get

$$2F\sin\theta=m\omega^2x$$

$$\Rightarrow \frac{2}{4\pi\varepsilon_0}\frac{q^2}{(d^2+x^2)}\sin\theta=m\omega^2x$$

$$\Rightarrow \frac{2}{4\pi\varepsilon_0}\frac{q^2}{(d^2+x^2)}\cdot\frac{x}{(d^2+x^2)^{1/2}}=m\omega^2x$$

$$\Rightarrow \ \omega = \left(\frac{1}{2\pi\varepsilon_0} \frac{q^2}{(d^2 + x^2)^{3/2} m} \right)^{1/2}$$

As, $x << d$

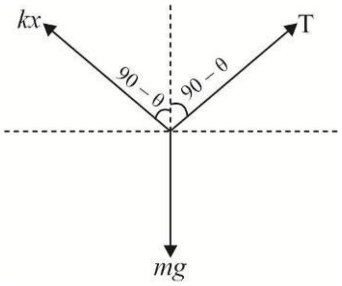
$$\therefore \ \omega = \left(\frac{1}{2\pi\varepsilon_0} \frac{q^2}{md^3} \right)^{1/2}$$

3.(A) $T = 2\pi\sqrt{\frac{l}{g}}$ $\qquad \frac{dT}{T} = \frac{1}{2} \times \frac{dl}{l}$

$$dT = \frac{1}{2} \times \frac{0.2}{100} \times 24 \times 3600 = 86.4$$

4.(C) Theory Based

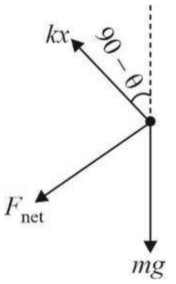
5.(C) Before string is cut



$$\frac{kx}{\sin(90+\theta)} = \frac{mg}{\sin(180-2\theta)}$$

$$\Rightarrow \ kx = \frac{mg}{2\sin\theta}$$

Just after string is cut



$$F_{net}^2 = (kx)^2 + (mg)^2 + 2(kx)(mg)\cos(90+\theta)$$

$$\Rightarrow \ F_{net} = \frac{mg}{2\sin\theta}$$

$$\Rightarrow \ a = \frac{F_{net}}{m} = \frac{g}{2\sin\theta}$$

6.(A) Mass of nucleus = Mass number $\times$ Mass of a nucleon

$$= A \times 1.67 \times 10^{-27} kg$$

Volume of nucleus

$$= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \times (1.3 \times 10^{-15} \times A^{1/3})^3$$

$$= \frac{4}{3}\pi(1.3)^3 \times 10^{-45} \times A$$

$\therefore$ Density of nucleus = mass/ volume

$$= \frac{A \times 1.67 \times 10^{-27}}{\frac{4}{3}\pi \times (1.3)^3 \times 10^{-45} \times A} \approx 10^{17}\, kg\, m^{-3}$$

7.(A) $\tan\alpha = \dfrac{B\sin(90+\beta)}{A + B\cos(90+\beta)}$

Since $\alpha = 90°$

$A + B\cos(90+\beta) = 0$

$A - 2A\sin\beta = 0$

$\sin\beta = \dfrac{1}{2}$

$\beta = 30°,$

$\Rightarrow \quad 90 + 30 = 120°$

8.(D) $\lambda \propto \dfrac{1}{\sqrt{E}}$

$\dfrac{\lambda_2}{\lambda_1} = \sqrt{\dfrac{E_1}{E_2}}$

$\dfrac{\lambda_2}{\lambda_1} = \dfrac{1}{2}$

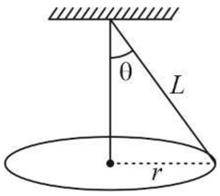
$\dfrac{E_1}{E_2} = \left(\dfrac{\lambda_2}{\lambda_1}\right)^2 = \dfrac{1}{4}$

$\Rightarrow \quad E_2 = 4E_1 = 4E$

Extra energy given = $4E - E = 3E$

9.(C) $\sin\theta = \dfrac{r}{L} = \dfrac{1}{2}$

$\theta = 30°$



Now, $\tan\theta = \dfrac{v^2}{rg}$

$\dfrac{1}{\sqrt{3}} = \dfrac{v^2}{rg}$

$v = \sqrt{\dfrac{rg}{\sqrt{3}}}$

10.(D) If an identical semicircle is joined to form a complete ring of mass $2m$, then I_A of complete ring by parallel axis theorem is $(2m)R^2 + 2mR^2 = 4mR^2$. Find if I is required moment of inertia then

$2I = 4mR^2$

$\Rightarrow \quad I = 2mR^2$

11.(A) The maximum electric field is $E_0 = 300\,V/m.$

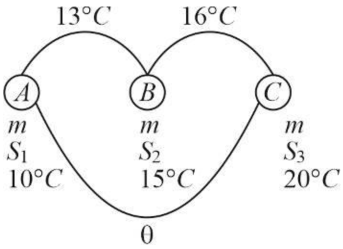
Along the *z*-direction

The maximum electric force on the electron is

$$F_e = qE_0 = (1.6\times10^{-19}C)\times(300\,V/m)$$

$$= 4.8\times10^{-17}\,N$$

12.(D)



When *A* and *B* are mixed

$$mS_1\times(13-10) = m\times S_2\times(15-13)$$

$$3S_1 = 2S_2 \qquad \dots \text{(i)}$$

When *B* and *C* are mixed

$$S_2\times1 = S_3\times4 \qquad \dots \text{(ii)}$$

When *C* and *A* are mixed

$$S_1(\theta-10) = S_3\times(20-\theta) \qquad \dots \text{(iii)}$$

By using equal (i),(ii) and (iii)

We get $\theta = \dfrac{140}{11}°C$

13.(A) Tension = mg

Strain $= \dfrac{\text{Stess}}{Y}$

$$\frac{mg}{AY}$$

14.(A) $\dfrac{T_L}{T_H - T_L} = \text{C.O.P.}$

$$\text{C.O.P.} = \frac{Q_H}{W} = \frac{253}{55}$$

$$Q_H = \frac{253}{55}\times55 = 253\,J/s$$

15.(C) The resistance of the circuit is $R = 50\,\Omega$

The reactance of the capacitor $= \dfrac{1}{\omega C}$

$$= \frac{1}{(2\pi\times50s^{-1})\,(100\times10^{-6}F)} = 31.8\,\Omega$$

The reactance of the inductor $= \omega L$

$$= (2\pi\times50s^{-1})\,(0.5\text{ henry}) = 157\,\Omega$$

The reactance of the circuit $= X = \dfrac{1}{\omega C} - L\omega$

$$= 31.8\,\Omega - 157\,\Omega = -125.2\,\Omega$$

Hence, the impedance $Z = \sqrt{R^2 + X^2}$

$$= \sqrt{(50\,\Omega)^2 + (125.2\,\Omega)^2} = 134.6\,\Omega$$

The rms current $= \dfrac{E_{rms}}{Z} = \dfrac{110V}{134.6\,\Omega} = 0.82\ A$

16.(B) Let mass of H_2 is m_1 and He is m_2

$\therefore \quad m_1 + m_2 = 10^{-2} kg = 10 \times 10^{-3} \qquad \ldots$(i)

Let P_1, P_2 are partial pressure of H_2 and He

$$P_1 + P_2 = 4.15 \times 10^5\, N/m^2$$

For the mixture

$$(P_1 + P_2)V = \left(\frac{m_1}{n_1} + \frac{m_2}{n_2}\right)RT$$

$$\Rightarrow \quad 4.15 \times 10^5 \times 2 \times 10^{-2} = \left(\frac{m_1}{2 \times 10^{-3}} + \frac{m_2}{4 \times 10^{-3}}\right) 8.31 \times 320$$

$$\Rightarrow \quad \frac{m_1}{2} + \frac{m_2}{4} = \frac{4.15 \times 2}{8.31 \times 320} = 0.00312 = 3.12 \times 10^{-3}$$

$$\Rightarrow \quad 2m_1 + m_2 = 12.48 \times 10^{-3} kg \qquad \ldots \text{(ii)}$$

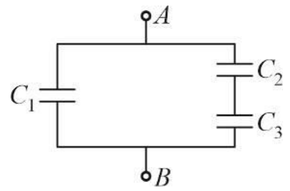
Solving (i) and (ii)

$$m_1 = 2.48 \times 10^{-3} kg \cong 2.5 \times 10^{-3} kg$$

17.(A) Resistance $\propto \dfrac{1}{\text{Power}}$. Thus, 40 W bulb has a high resistance. Because of which there will be more potential drop across 40 W bulb. Thus 40 W bulb will glow brighter.

18.(A) It is equivalent to

$$C = C_1 + \frac{C_2 C_3}{C_2 + C_3}$$



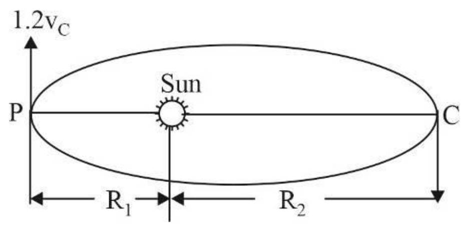
$$C = \frac{A_1 K_1 \varepsilon_0}{d_1 + d_2} + \frac{\dfrac{A_2 K_2 \varepsilon_0}{d_1} \cdot \dfrac{A_2 K_3 \varepsilon_0}{d_2}}{\dfrac{A_2 K_2 \varepsilon_0}{d_1} + \dfrac{A_2 K_2 \varepsilon_0}{d_2}}$$

$$= \frac{A_1 K_1 \varepsilon_0}{d_1 + d_2} + \frac{A_2^2 K_2 K_3 \varepsilon_0^2}{A_2 K_2 \varepsilon_0 d_2 + A_2 K_3 \varepsilon_0 d_1} \quad = \frac{A_1 K_1 \varepsilon_0}{d_1 + d_2} + \frac{A_2 K_2 K_3 \varepsilon_0}{K_2 d_2 + K_3 d_1}$$

19.(A) The comet is moving along an elliptical path with the sun situated at one of the foci of ellipse. Q is the farthest position of the comet.

$$\frac{m v_c^2}{R_1} = G \frac{Mm}{R_1^2}$$

$$v_c^2 = G \frac{M}{R_1} \qquad \ldots \text{(i)}$$



Energy and angular momentum at P

$$E_P = \frac{1}{2}mv_p^2 - G\frac{Mm}{R_1}$$

$$E_P = \frac{1}{2}m(1.2v_c)^2 - \frac{GMm}{R_1}$$

$$= \frac{1}{2}m(1.2)^2\frac{GM}{R_1} - \frac{GMm}{R_1}$$

$$E_P = -0.28\frac{GMm}{R_1} \qquad \ldots\text{(ii)}$$

and $L_P = mv_P R_1$ …(iii)

At Q,

$$E_Q = \frac{1}{2}mv_Q^2 - \frac{GMm}{R_2} \qquad \ldots\text{(iv)}$$

$$L_Q = mv_Q R_2 \qquad \ldots\text{(v)}$$

Energy and momentum should be conserved

$$L_1 = L_2$$

$$mv_P R_1 = mv_Q R_2 \Rightarrow \quad v_Q = v_P\frac{R_1}{R_2}$$

$$= -0.28\frac{GMm}{R_1} = \frac{1}{2}mv_P^2\left(\frac{R_1}{R_2}\right)^2 - \frac{GMm}{R_2} = \frac{1}{2}\times(1.2)^2\times\frac{GMm}{R_1}\times\left(\frac{R_1}{R_2}\right)^2 - \frac{GMm}{R_2}$$

$$= -0.28 = 0.72\left(\frac{R_1}{R_2}\right)^2 - \frac{R_1}{R_2}$$

$$= 0.72X^2 - X + 0.28 = 0 \qquad \text{Where } \frac{R_1}{R_2} = X$$

$$X = \frac{1 \pm 0.44}{1.44} = \frac{R_1}{R_2}$$

Taking -ve sign, we get

$$R_2 = \frac{1.44}{0.56} = 2.57R_1$$

20.(A) $V = \frac{Kq}{R}$

$\therefore \quad Kq = VR$

$$E = \frac{Kq}{r^2} = \frac{VR}{r^2}$$

SECTION-2

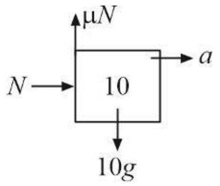
1.(20) Applying *NL* in horizontal direction

$N = 10$ a … (i)

Applying *NL* in vertical direction

$10g = \mu N$ … (ii)

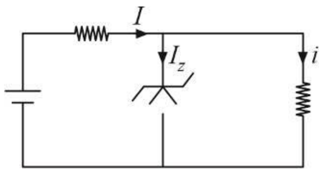
$10g = \mu\, 10\, a$ from (i) and (ii)



$\therefore \qquad a = \dfrac{g}{\mu} = 20\ m/s^2$

2.(130)

$i = \dfrac{10}{10 \times 10^3} = 1\ mA$

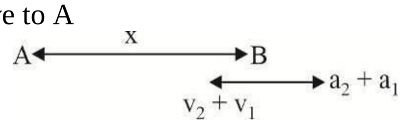


$I = \dfrac{14}{10^3} = 14\ mA$

$I_z = 14 - 1 = 13\ mA$

$P = I_z V_z = 130\ mW$

3.(8) Relative to A



$$x = \frac{(V_2 + V_1)^2}{2(a_1 + a_2)} = \frac{(5+3)^2}{2(2+2)} = \frac{64}{8} = 8\,m$$

4.(300) $E < 10^6$; $\qquad \dfrac{10^3}{d} = 10^6$

$d > 10^{-3} m^2$; $\qquad C = \dfrac{k\varepsilon_0 A}{d}$

$d = \dfrac{k\varepsilon_0 A}{C} > 10^{-3}$; $\qquad A > \dfrac{10^{-3} \times C}{k\varepsilon_0}$

$A > \dfrac{10^{-3} \times 50 \times 10^{-12}}{6\pi \times \left(\dfrac{1}{36\pi} \times 10^{-9}\right)}$; $\qquad \boxed{A > 300 mm^2}$

5.(2) For equilibrium

$Mg = IlB$

$I = \dfrac{Mg}{lB} = \dfrac{40 \times 10^{-3} \times 10}{50 \times 10^{-2} \times 0.4} = 2A$

6.(60) $\dfrac{I}{2}\cos^2\phi = \dfrac{I}{8}$

$\cos\phi = \dfrac{1}{2}$

$\phi = 60°$

7.(30) For lens,

$\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

$\dfrac{1}{v} - \dfrac{1}{-15} = \dfrac{1}{10} \qquad \Rightarrow \qquad v = +30\ cm$

Hence it is object for mirror

$u = -15\ cm$

$\dfrac{1}{v} + \dfrac{1}{-15} = \dfrac{1}{-10} \qquad \Rightarrow \qquad v = -30\ cm$

Now for second time it again passes through lens

$u = -15\ cm$

$v = ?$; $f = 10\ cm$

$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10} \quad \Rightarrow \quad v = +30\ cm$

Hence final image will form at a distance 30 *cm* from the lens towards left.

8.(75) $E_{MQ} + E_{PM} = E_{PQ}$

corner $\rightarrow \frac{B\omega l^2}{2} = 100$

$E_{MQ} + \frac{B\omega\left(\frac{l}{2}\right)^2}{2} = \frac{B\omega l^2}{2} \quad \Rightarrow \quad E_{MQ} = \frac{3}{8}B\omega l^2 = \frac{3}{4} \times 100V = 75V$

9.(4) $\vec{\tau} = \vec{M} \times \vec{B}$

$\tau = MB\sin 90°$

$= i \times (\text{side})^2 \times B \times 1 = 0.2 \times 0.1 \times 0.1 \times 20 \times 10^{-3} = 4 \times 10^{-5}\ Nm$

$x = 4$

10.(4) $y = y_1 + y_2 + y_3$

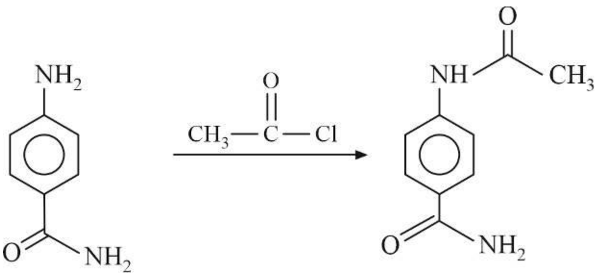
$y = 4A(\sin\omega t)(\cos kx)$

$y_{\max} = 4A$

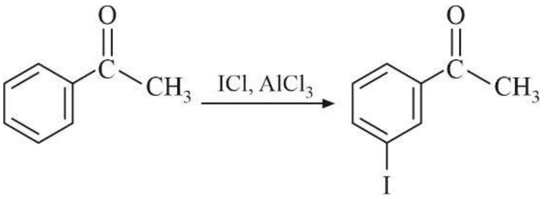
CHEMISTRY

SECTION-1

1.(D)  has 2π electrons in cyclic conjugation, follows Huckel's rule.

2.(B) 

3.(D) (Meta directing)

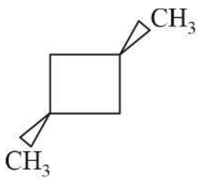


4.(C) Lanthanoid contraction is not valid for group 3. Hence atomic size of Y & LA are not equal.

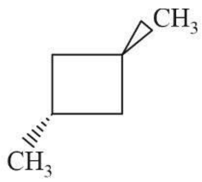
5.(C) 15^{th} group is also called Pnictogens, i.e. N, P, As, Sb, Bi

6.(D) $[Co(NC)_5(CN)]^{3-}$ will not show geometrical isomerism.

7.(A)



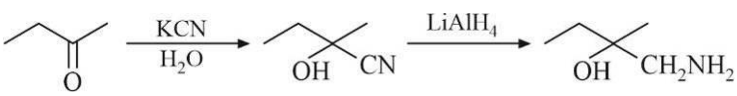
Optically inactive due to symmetry



Optically inactive due to symmetry

8.(A) Mn in $KMnO_4$ is +7 oxidized with d^0 configuration hence no d-d transition is possible. It is coloured due to LMCT.

9.(A) O^{2-} and O_2^{2-} both don't have unpaired electrons.

10.(A) 

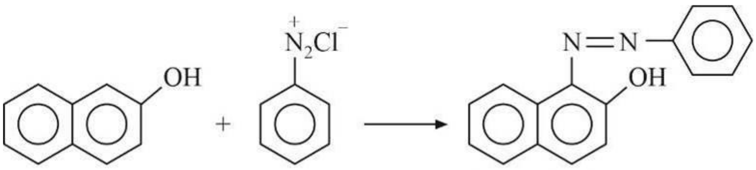
11.(A) As we know, the spectrochemical series is :

$$F^- < Br^- < SCN^- < Cl^- < S^{2-} < F^- < OH^- < C_2H_4^{2-} < H_2O < NCS^- < edta^{4-}$$

$$< NH_3 < en < CN^- < CO$$

Thus order of CFSE for given complexes will be

$$[Co(H_2O)_6]^{3+} < [Co(NH_3)_6]^{3+} < [Co(en)_3]^{3+} < [Co(CN)_6]^{3-}$$

12.(A) 

(orange red coloured dye)

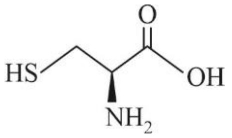
13.(C) Dipole dipole interaction energy $\propto \frac{1}{r^3}$

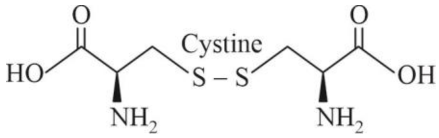
14.(C) $\pi \propto i$

$i_{BaCl_2} = 3$

$i_{NaCl} = 2$

$\underset{\text{Hypertonic}}{\pi_{BaCl_2}} > \underset{\text{Hypotonic}}{\pi_{NaCl}}$

15.(D) Cysteine 



16.(B) $CH_3COOH \xrightarrow{\text{(i) } Ca(OH)_2} \left(CH_3-\overset{\overset{\displaystyle O}{||}}{C}-O\right)_2 Ca \xrightarrow{\Delta} CH_3COCH_3$

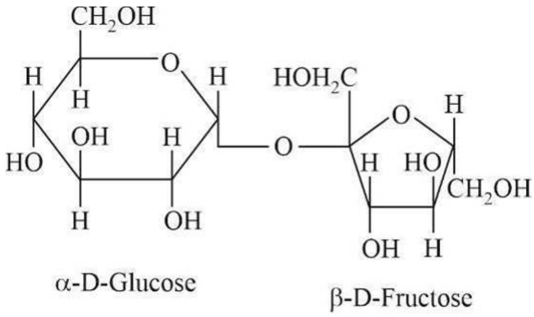
$CH_3COCl \xrightarrow{H_2, Pd/BaSO_4} CH_3CHO$

$CH_3CN \xrightarrow[H_2O]{SnCl_2, HCl} CH_3CHO$

$CH_3COCl \xrightarrow{(CH_3)_2CuLi} CH_3COCH_3$

17.(B) F atom has less -ve electron gain enthalpy than Cl atom because additional electron faces more repulsion in 2p as compared to 3p orbital.

18.(B) The two monosaccharide are held together by a glycosidic linkage between C_1 of α-D-Glucose and C_2 of β-D-fructose. Since the reducing groups of glucose and fructose are involved in glycosidic bond formation sucrose is a non-reducing sugar.



Sucrose

19.(D) $[Ti(H_2O)_6]^{3+}$

$Ti^{3+} \Rightarrow d^1 \rightarrow$ d-d transition possible $\Rightarrow$ coloured compound

$[Sc(H_2O)_6]^{3+}$

$Sc^{3+} \Rightarrow d^0 \Rightarrow$ d-d transition is not possible $\Rightarrow$ colourless compound

20.(A) Fact

SECTION – 2

1.(5) $M^{+2} + 4L \rightleftharpoons [ML_4]^{2+} \quad K_f = 2\times10^{10}$

Then $[ML_4]^{2+} \rightleftharpoons M^{+2} + 4L \quad K_{diss} = \dfrac{1}{2\times10^{10}} = 5\times10^{-11}$

2.(1) $E_{cell} = E^{\circ}_{cell} - \dfrac{0.059}{2}\log\dfrac{[Zn^{+2}]}{[Cu^{2+}]} = 1.09$

3.(5) Number of moles of $Na^+ = \dfrac{6.9}{23} = 0.3$

Moles of $Na_3PO_4 = \dfrac{0.3}{3} = 0.1$

$\text{Molarity} = \dfrac{0.1\times1000}{200} = 0.5 = 5\times10^{-1}M$

4.(4) We know that at equilibrium $\dfrac{-d[[PtCl_4]^{2-}]}{dt} = 0$

Thus $4.8\times10^{-8}[[PtCl_4]^{2-}] = 1.2\times10^{-3}[Pt(H_2O)Cl_3]^-[Cl^-]$

$$K_c = \frac{[[Pt(H_2O)Cl_3]^-][Cl^-]}{[[PtCl_4]^{2-}]} = \frac{4.8\times10^{-8}}{1.2\times10^{-3}} = 4\times10^{-5}$$

5.(3) Number of moles of chloroalkane $= \dfrac{3.06}{M_o} = \dfrac{873}{22400}$

$M_o = 78.5$

From formula $C_nH_{2n+1}Cl$

$12n + 2n + 1 + 35.5 = 78.5$

$n = 3$

6.(64) Mass of S $= \dfrac{2.33}{233}\times32 = 0.32$

% of S $= \dfrac{0.32}{0.5}\times100 = 64\%$

7.(5) $E = \dfrac{hc}{\lambda} = \dfrac{6.63\times10^{-34}\times3\times10^8}{1000\times10^{-9}} = 2\times10^{-19}J$

Work function (W), $h\nu_0 = 6.63\times10^{-34}\times\dfrac{10^{14}}{6.63} = 10^{-20}$

$K.E. = \dfrac{1}{2}mv^2 = E - W$

$\dfrac{1}{2}\times9\times10^{-31}\times v^2 = 2\times10^{-19} - 10^{-20} = 19\times10^{-20}$

$v^2 = \dfrac{38\times10^{-20}}{9\times10^{-31}} = \dfrac{38}{9}\times10^{11} = \dfrac{380}{9}\times10^{10} = 42.2\times10^{10}$

$v = 6.5\times10^5 m/sec$

8.(40) $H_2O(l) \longrightarrow H_2O(g)$

$\Delta H_{vap} = 43\ kJ/mol$

$q = \Delta H_{vap} = 43\ kJ/mol$

$w = -P\Delta V = -P[V_{H_2O(g)} - V_{H_2O(l)}]$

Volume of $V_{H_2O(l)}$ is negligible

$w = -PV_{H_2O(g)} = -nRT = -\dfrac{1\times 8.3\times 373}{1000} = -3.095\ kJ$

$\Delta U = q + w = 43 - 3.095 = 39.90\ kJ/mol$

9.(271) Molality $= \dfrac{62}{62\times 1} = 1$ molal

$\Delta T_f = m \times K_f = 1.86$

$\Delta T_f = 1.86 = T_f^\circ - T_f$

$T_f = T_f^\circ - 1.86 = 271.14 \approx 271$

10.(2)

	A	+	B	$\rightleftharpoons$	C	+	D
Initial	1		1		1		1
at equilibrium	$1-x$		$1-x$		$1+x$		$1+x$

$K_C = \dfrac{(1+x)^2}{(1-x)^2} = 144$

$\dfrac{1+x}{1-x} = 12 \quad \Rightarrow \quad 1 + x = 12 - 12x$

$13x = 11$

$x = \dfrac{11}{13}$

$[D] = 1 + \dfrac{11}{13} = \dfrac{24}{13} = 1.85 \approx 2$

MATHEMATICS

SECTION-1

1.(C) $A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 2 & 0 & 0 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 2 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 1 \\ 2 & 0 & 0 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 1 \\ 2 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 1 \\ 2 & 0 & 0 \end{bmatrix}$

Similarly, $A^{2022} = \begin{bmatrix} 1 & 0 & 0 \\ 4042 & 1 & 1 \\ 2 & 0 & 0 \end{bmatrix}$

$A^{2010} = \begin{bmatrix} 1 & 0 & 0 \\ 4018 & 1 & 1 \\ 2 & 0 & 0 \end{bmatrix}$

$\Rightarrow \quad A^{2022} - A^{2010} = \begin{bmatrix} 0 & 0 & 0 \\ 24 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$A^{20} - A^{8} = \begin{bmatrix} 1 & 0 & 0 \\ 38 & 1 & 1 \\ 2 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 14 & 1 & 1 \\ 2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 24 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

2.(D) $I = \int_0^{20} e^{x-[x]}\,dx$

$= 20\int_0^1 e^{x-[x]}\,dx \qquad (e^{x-[x]}$ has period 1)

$= 20\int_0^1 e^x\,dx = 20\left(e^x\right)_0^1 = 20(e-1)$

3.(A) $\tan\theta = \dfrac{1-0}{2-1} = 1$

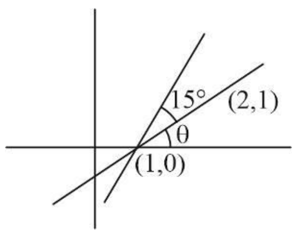
$\theta = 45°$

Slope of line in its new position = $\tan 60° = \sqrt{3}$

Equation of line in its new position is

$y - 0 = \sqrt{3}\,(x-1)$

$\Rightarrow \quad \sqrt{3}x - y - \sqrt{3} = 0$



4.(D) $P(A \cup B) \geq \max\{P(A), P(B)\} = \dfrac{2}{3}$

$P(A \cap B) = P(A) + P(B) - P(A \cup B) \geq P(A) + P(B) - 1 = \dfrac{1}{6}$

and $P(A \cap B) \leq \min\{P(A), P(B)\} = \dfrac{1}{2} \Rightarrow \dfrac{1}{6} \leq P(A \cap B) \leq \dfrac{1}{2}$

Also, $P(A \cap B') = P(A) - P(A \cap B) \leq \dfrac{1}{2} - \dfrac{1}{6} = \dfrac{1}{3}$

Lastly $P(A' \cap B) = P(B) - P(A \cap B)$

$\dfrac{3}{2} - \dfrac{1}{2} \leq P(A' \cap B) \leq \dfrac{1}{3} - \dfrac{1}{6} \Rightarrow \dfrac{1}{6} \leq P(A' \cap B) \leq \dfrac{1}{2}$

5.(A) $\lim\limits_{x \to 2} \sum\limits_{n=2}^{10} \dfrac{[(n+2)x+3]-[nx+3]}{[(n+2)x+3][nx+3]}$

$\lim\limits_{x \to 2} \sum\limits_{n=2}^{10} \dfrac{1}{nx+3} - \dfrac{1}{(n+2)x+3}$

$\lim\limits_{x \to 2} \left(\dfrac{1}{2x+3} + \dfrac{1}{3x+3} - \dfrac{1}{11x+3} - \dfrac{1}{12x+3} \right)$

$\dfrac{1}{7} + \dfrac{1}{9} - \dfrac{1}{25} - \dfrac{1}{27} = \dfrac{836}{4725}$

6.(A) $T_n = \tan^{-1} \dfrac{3}{1+n^2+n-2} = \tan^{-1} \dfrac{(n+2)-(n-1)}{1+(n-1)(n+2)}$

$T_n = \tan^{-1}(n+2) - \tan^{-1}(n-1)$

$T_1 = \tan^{-1} 3 - \tan^{-1} 0$

$T_2 = \tan^{-1} 4 - \tan^{-1} 1$

$T_3 = \tan^{-1} 5 - \tan^{-1} 2$

$T_4 = \tan^{-1} 6 - \tan^{-1} 3$

$\vdots \qquad \vdots \qquad \vdots$

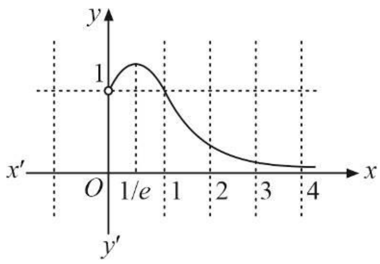
$T_n = \tan^{-1}(n+2) - \tan^{-1}(n-1)$

On adding these equations

$\sum\limits_{n=1}^{\infty} T_n = \dfrac{3\pi}{2} - \tan^{-1} 0 - \tan^{-1} 1 - \tan^{-1} 2$

$= \dfrac{3\pi}{2} - 0 - \dfrac{\pi}{4} - \tan^{-1} 2 = \dfrac{5\pi}{4} - \tan^{-1} 2 = \dfrac{3\pi}{4} + \cot^{-1} 2$

7.(D) $f(x)=\left(\frac{1}{x}\right)^x \Rightarrow f'(x)=\left(\frac{1}{x}\right)^x\left(\ln\frac{1}{x}-1\right)$



$f'(x)=0 \Rightarrow \ln\frac{1}{x}=1$

$\Rightarrow \frac{1}{x}=e$

$\Rightarrow x=\frac{1}{e}$

Also for $x<\frac{1}{e}$, $f'(x)$ is positive and for $x>\frac{1}{e}$, $f'(x)$ is negative

Hence, $x=\frac{1}{e}$ is point of maxima

Therefore, the maximum value of function is $e^{1/e}$

Also, $\lim\limits_{x\to 0}\left(\frac{1}{x}\right)^x = e^{\lim\limits_{x\to 0} x\ln\left(\frac{1}{x}\right)} = e^{-\lim\limits_{x\to 0} x\ln x} = e^0 = 1$

$\lim\limits_{x\to 0}\left(\frac{1}{x}\right)^x = 1$

8.(B) Let $I=\int_{-\pi/4}^{\pi/4}\frac{\tan^2 x}{1+e^x}\,dx$ … (i)

Using property $\int_a^b f(x)\,dx=\int_a^b f(a+b-x)\,dx$

$I=\int_{-\pi/4}^{\pi/4}\frac{\tan^2 x}{1+e^{-x}}\,dx$ … (ii)

By adding (i) and (ii)

$2I=\int_{-\pi/4}^{\pi/4}\tan^2 x\,dx$

$2I=\int_{-\pi/4}^{\pi/4}(\sec^2 x-1)\,dx$

$2I=\left(\tan x-x\right)_{-\pi/4}^{\pi/4}$

$2I=\left(1-\frac{\pi}{4}\right)-\left(-1+\frac{\pi}{4}\right)$

$2I=2\left(1-\frac{\pi}{4}\right)$

$I=1-\frac{\pi}{4}$

9.(B) $p = \{1, 2, 3, 4, 5, 6\}$

$q = \{1, 2, 3, 4, 5, 6\}$

$$D = \begin{vmatrix} 2 & p & 6 \\ 1 & 2 & q \\ 1 & 1 & 3 \end{vmatrix} = (q-3)(p-2)$$

$$D_1 = \begin{vmatrix} 8 & p & 6 \\ 5 & 2 & q \\ 4 & 1 & 3 \end{vmatrix} = (p-2)(4q-15)$$

$$D_2 = \begin{vmatrix} 2 & 8 & 6 \\ 1 & 5 & q \\ 1 & 4 & 3 \end{vmatrix} = 0$$

$$D_3 = \begin{vmatrix} 2 & p & 8 \\ 1 & 2 & 5 \\ 1 & 1 & 4 \end{vmatrix} = p-2$$

For unique solution

$$D \neq 0$$

$$\Rightarrow \quad p \neq 2,\ q \neq 3$$

$$p_1 = \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$$

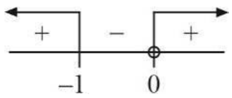
For infinite solutions

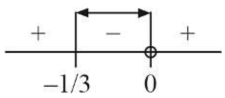
$$D = D_1 = D_2 = D_3 = 0$$

$$\Rightarrow \quad p = 2 \text{ and } q = \{1, 2, 3, 4, 5, 6\} \Rightarrow \quad p_2 = \frac{1}{6} \times 1 = \frac{1}{6} \quad \Rightarrow \quad \frac{p_1}{p_2} = \frac{25/36}{1/6} = \frac{25}{6}$$

10.(C) $\dfrac{1+2x}{x} \geq 1$ or $\dfrac{1+2x}{x} \leq -1$

$\Rightarrow \dfrac{1+x}{x} \geq 0$ $\qquad \Rightarrow \dfrac{1+3x}{x} \leq 0$





$x \in (-\infty, -1] \cup (0, \infty)$ ... (i) $\qquad x \in \left[-\dfrac{1}{3}, 0\right)$ ... (ii)

Taking union of (i) and (ii) ; we get $x \in (-\infty, -1] \cup \left[-\dfrac{1}{3}, 0\right) \cup (0, \infty)$

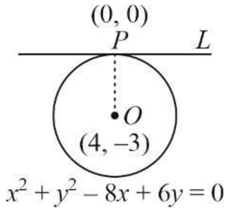
or $\dfrac{|1+2x|}{|x|} \geq 1$

11.(C) $SD = \left|\dfrac{(\vec{a}_1 - \vec{a}_2)(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right| = \dfrac{1}{\sqrt{38}}$

$$\frac{\begin{vmatrix} 1 & \lambda-2 & 2 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix}}{\sqrt{38}} = \frac{1}{\sqrt{38}} \Rightarrow \lambda = \frac{13}{5}, 3$$

12.(C) Equation of variable circle will be

$$S + \lambda L = 0$$



S is a point circle with centre at (0, 0) and r = 0

$$S : (x-0)^2 + (y-0)^2 = 0$$

$$L : (y-0) = \frac{4}{3}(x-0) \quad \Rightarrow \quad 4x - 3y = 0$$

[$\therefore$ *L* is perpendicular to *OP*]

Equation of family of circle is

$$x^2 + y^2 + \lambda(4x - 3y) = 0$$

Circle which passes through (–1, 2)

$$1^2 + 2^2 + \lambda(-4-6) = 0 \quad \Rightarrow \quad 5 - 10\lambda = 0 \quad \Rightarrow \quad \lambda = \frac{1}{2}$$

Equation of required circle will be

$$x^2 + y^2 + \frac{1}{2}(4x - 3y) = 0 \quad \Rightarrow \quad 2x^2 + 2y^2 + 4x - 3y = 0$$

Radius $= \sqrt{1 + \dfrac{9}{16}} = \dfrac{5}{4}$

13.(A) Let the number is *abcdef* then $a + c + e = 12 = b + d + f$ as $0+1+2+5+7+9 = 24$

Case-I $\{a, c, e\} = \{0, 5, 7\}$

$\{b, d, f\} = \{1, 2, 9\}$

i.e., $2 \times 2!3! = 24$

Case-II $\{a, c, e\} = \{1, 2, 9\}$

$\{b, d, f\} = \{0, 5, 7\}$

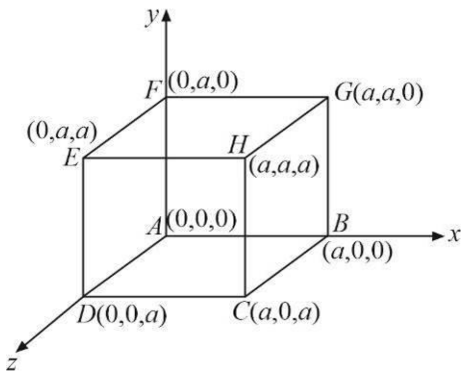
i.e., $3! \times 3 = 36$

$\therefore$ Total ways $= 24 + 36 = 60$

14.(B) $(1-i)^2 = 1 + i^2 - 2i = -2i$

$(1-i)^{50} = -2^{25} i$

15.(A)



Let length side of cube is 'a' cm.

Direction ratios of diagonal AH are a, a, a

Direction ratios of diagonal CF are $-a, a, -a$

Angle between AH and CF is

$$\cos\theta = \left|\frac{-a^2 + a^2 - a^2}{\sqrt{3}a\,\sqrt{3a}}\right|$$

$$\cos\theta = \frac{1}{3}$$

16.(B) $f(k^2 + 2k + 5) > f(k + 11)$

$\Rightarrow \; k^2 + 2k + 5 < k + 11$

$k^2 + k - 6 < 0$

$(k+3)(k-2) < 0$

$\Rightarrow \; k \in (-3, 2)$

17.(A) Equation of circle is

$$\left(x - \frac{P}{2}\right)^2 + (y-0)^2 = P^2 \qquad \ldots\text{(i)}$$

and parabola $y^2 = 2px$ ...(ii)

from equation (i) and (ii)

$$4x^2 + 4px - 3p^2 = 0 \;\Rightarrow\; x = \frac{P}{2}, \frac{-3P}{2}$$

$\therefore \; y = \pm P$ s

$\therefore$ Point of intersection are

$$\left(\frac{P}{2}, P\right) \& \left(\frac{P}{2}, -P\right)$$

18.(A) $2x^2 \dfrac{dy}{dx} + e^y + 2x = 0$

$$\Rightarrow \qquad e^{-y}\frac{dy}{dx} + \frac{e^{-y}}{x} + \frac{1}{2x^2} = 0$$

Let $\qquad e^{-y} = z$

$$\Rightarrow \qquad -e^{-y}\frac{dy}{dx} = \frac{dz}{dx}$$

$\Rightarrow \quad \dfrac{dz}{dx} + z\left(-\dfrac{1}{x}\right) = \dfrac{1}{2x^2}$

Its I.F. $= e^{\int -\frac{1}{x}dx} = \dfrac{1}{x}$

Solution $\quad z(\text{I.F.}) = \int \dfrac{1}{x} \cdot \dfrac{1}{2x^2}\, dx + C$

$$\frac{z}{x} = -\frac{1}{4x^2} + C$$

$\therefore \quad e^{-y} = -\dfrac{1}{4x} + Cx$

Now as given $x = 1$, $y = 0$

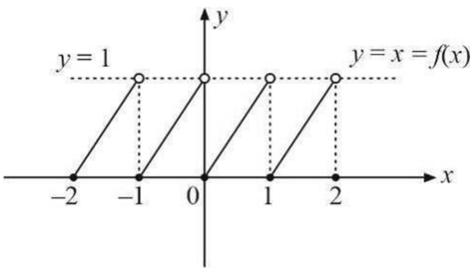
$\Rightarrow \quad C = \dfrac{5}{4}$

$$e^{-y} = -\frac{1}{4x} + \frac{5x}{4}$$

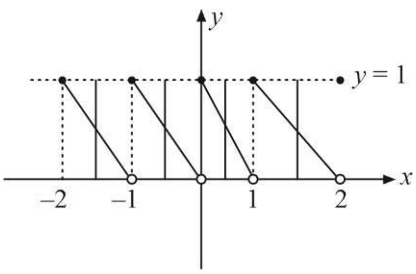
$\therefore \quad y = \log_e\left(\dfrac{4x}{5x^2 - 1}\right)$

Now when $x = 2$ then $y = \log_e\left(\dfrac{8}{19}\right)$

19.(D) $f(x) = \{x\}$ and $g(x) = -\{x\}$

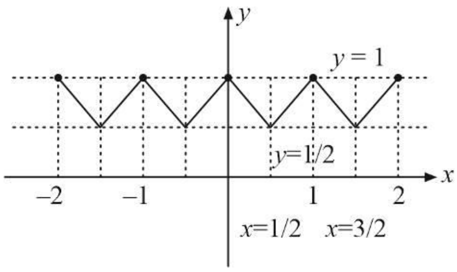


and $\quad y = g(x) = 1 - \{x\}$



$\{x\} = 1 - \{x\} \Rightarrow \{x\} = \dfrac{1}{2}$

$x = I + \dfrac{1}{2}$



Continuous in [–2, 2] but not differentiate at more than four point in (–2, 2).

20.(D) $2\cos\frac{\pi}{8}\cos\left(\frac{2\pi}{8}\right)\cos\left(\frac{3\pi}{8}\right)\cos\left(\pi-\frac{3\pi}{8}\right)\cos\left(\pi-\frac{2\pi}{8}\right)\cos\left(\pi-\frac{\pi}{8}\right)$

$=-2\cos^2\frac{\pi}{8}\cos^2\left(\frac{2\pi}{8}\right)\cos^2\left(\frac{3\pi}{8}\right)$

$=-\frac{1}{4}\left(1+\cos\frac{\pi}{4}\right)\left(1+\cos\frac{2\pi}{4}\right)\left(1+\cos\frac{3\pi}{4}\right) = -\frac{1}{4}\left(1+\frac{1}{\sqrt{2}}\right)\cdot 1\cdot\left(1-\frac{1}{\sqrt{2}}\right)-\frac{7}{9}\left(1-\frac{1}{2}\right)=-\frac{1}{8}$

SECTION - 2

1.(2) Projection of $\hat{i}+2j+k$ on $(2-\lambda)\hat{i}+(\lambda-2)j-2k$

$=\frac{(2-\lambda)+2(\lambda-2)-2}{\sqrt{(2-\lambda)^2+(\lambda-2)^2+4}}=1$

$\Rightarrow \quad (\lambda-4)^2=2(\lambda-2)^2+4$

$\lambda^2-8\lambda+16=2\lambda^2-8\lambda+8+4$

$\lambda^2=4\Rightarrow|\lambda|=2$

$|\lambda|=2$

2.(306) Here $\vec{n}_1\times\vec{n}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 4 & 1\\ 3 & -4 & 0\end{vmatrix}=4\hat{i}+3\hat{j}+2\hat{k}$

Now $SD=\left|\frac{(2\lambda\hat{i}+3\hat{j}-12\hat{k})\cdot(4\hat{i}+3\hat{j}+2\hat{k})}{\sqrt{16+9+144}}\right|=13$

$\Rightarrow |8\lambda+9+144|=169\Rightarrow \lambda=\frac{16}{8},\frac{-322}{8}$

$\Rightarrow 8\left|\sum_{\lambda\in S}\lambda\right|=8\left|\frac{16\lambda}{8}-\frac{322}{8}\right|=306$

3.(2) for $\quad n=1,\ \frac{2i}{(1-i)^{-1}}=2i(1-i)\neq \text{Real}$

$n=2,\frac{(2i)^2}{(1-i)^0}=-4=\text{Real No.}$

$\Rightarrow \quad n=2$

4.(1) $C_2 = 12 = (a_1 + d)\,(b \cdot r) = (a_1 + 1)(2b_1)$ … (i)

and $C_3 = 32 = (a_1 + 2)\,4b_1 = 32$ … (ii)

From (i) and (ii) $\dfrac{4b_1(a_1+2)}{(a_1+1)(2b_1)} = \dfrac{32}{12}$

$3a_1 + 6 = 4a_1 + 4 \Rightarrow a_1 = 2$

$\therefore \quad (a_1 + 1)2b_1 \Rightarrow 12$

$\Rightarrow \quad 3 \times 2b_1 = 12$

$b_1 = 2$

$\therefore \quad C_{10} = (a_1 + 9d)(b_1 \cdot r^{n-1}) = (2+9)\,2 \cdot 2^9 = 11.2^{10}$

$= \alpha \cdot 2^{\beta}$ where $\alpha\beta$ a prime number

$\therefore \quad |\alpha - \beta| = 1$

5.(16) $\det(5A^4B^3) = 5^3\,2^4\,3^3 = 2^4\,3^3\,5^3$

Division of the form $= 4k + 2 = 2 \cdot (2k+1)$

= 2 × (odd No)

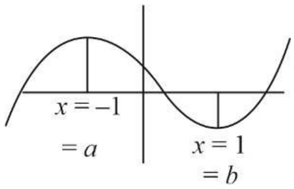
= Number of odd division

= (3 + 1) (3 + 1) = 16

6.(5) $f(x) = 3x^2 - 3 = 0$

$\Rightarrow \quad x = -1, 1$

$A =$ area $= 2\int_{-1}^{0} x^3 - 3x\,dx = 2\left(\dfrac{x^4}{4} - \dfrac{3x^2}{2}\right)\Bigg|_{7}^{0} = 2\left(\dfrac{1}{2} - \dfrac{1}{4}\right)$



$A = \dfrac{5}{2} = 2 \times \dfrac{5}{4} = \dfrac{5}{2}$ sq. unit $\quad \Rightarrow \quad 2A = 5$

7.(5) $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 2 & 1 \end{vmatrix} = 4\hat{i} + 5\hat{j} - 2\hat{k}$

Hence equation of L is $\vec{r} = \sigma(-4\hat{i} + 5\hat{j} - 2\hat{k})$

From intersection of L and L_1

$1 + \lambda = -4\sigma$

$-11 + 2\lambda = 5\sigma \Rightarrow \lambda = 3$

$-7 + 3\lambda = -2\sigma \qquad \sigma = -1$

Hence point $P \equiv (4, -5/2)$

Any point of L_2 is $Q(2\mu - 1, 2\mu, (\mu - 1))$

dr's of PQ is $(-5 + 2\mu, 2\mu + 5, \mu - 1)$

$\because$ PQ is perpendicular to L_2

$\therefore\ 2(-5 + 2\mu) + 2(2\mu + 5) + (\mu - 1) = 0 \Rightarrow \mu = \dfrac{1}{9}$

Hence $9(\alpha+\beta+x)=9\left(\frac{5}{9}\right)=5$

8.(28) $\text{Mean}(\bar{x})=\frac{2+3+7+x+y}{5}=5$

$\therefore \quad x+y=13 \quad \ldots \text{(i)}$

and variance $=\frac{\sum(x_c)^2}{N}-(\bar{x})^2$

$10=\frac{2^2+3^2+7^2+x^2+y^2}{5}-5^2$

$\Rightarrow \quad x^2+y^2=113$

$\therefore \quad xy=\frac{1}{2}[(x+y)^2-(x^2+y^2)]=\frac{1}{2}(13^2-113)=28$

9.(4455)

Let the three digit numbers are 308, 319, 330, 314, …495

Now $\quad T_n=a+(n-1)d$

$\Rightarrow \quad 495=308+(n-1)\times 11$

$\Rightarrow \quad (n-1)=\frac{495-308}{11}=17$

$\therefore \quad n=18$

So sum $=308+319+330+\ldots=\frac{18}{2}(308+495)=7227$

Now number containing 1 and 2

341, 451, 319, 418

352, 462, 429

Required sum = 7227 – (341 + 451 + 319 + 418 + 352 + 462 + 429) = 4455

10.(2) As $\alpha,\ \beta$ common root of equations

$$\therefore \quad \left.\begin{array}{l}\alpha^2-\lambda\alpha+2=0\\ 3\alpha^2-10\lambda\alpha+27=0\end{array}\right\} \Rightarrow \quad \frac{\alpha^2}{-7\lambda}=\frac{-\alpha}{21}=\frac{1}{-7\lambda}$$

$\Rightarrow \quad \alpha=\frac{\lambda}{3}=\frac{1}{\lambda} \quad \Rightarrow \quad \lambda^2=9 \quad \Rightarrow \quad \lambda=\pm 3$

As given $\lambda<0, \lambda=-3$

Now $\quad x^2+3x+2=0 \quad$ and $\quad 3x^2+30x+27=0$

$\swarrow \qquad\qquad x^2+10x+9=0$

Roots = –1, –2 $\qquad \swarrow \quad \searrow$

Roots = –9, –1

$\therefore \quad \alpha-1, \beta=-2, \gamma=-9$

$\therefore \quad \left(\frac{\beta\gamma}{\lambda^2}\right)=\frac{18}{9}=2$